Math 2D Quiz 3 Morning - February 4th

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1. (a) Sketch the curve $x^2 - y^2 - z^2 - 4x - 2y + 4z - 5 = 0$.

~~It is okay if your sketch is rough, but~~ be sure to clearly note any translations.

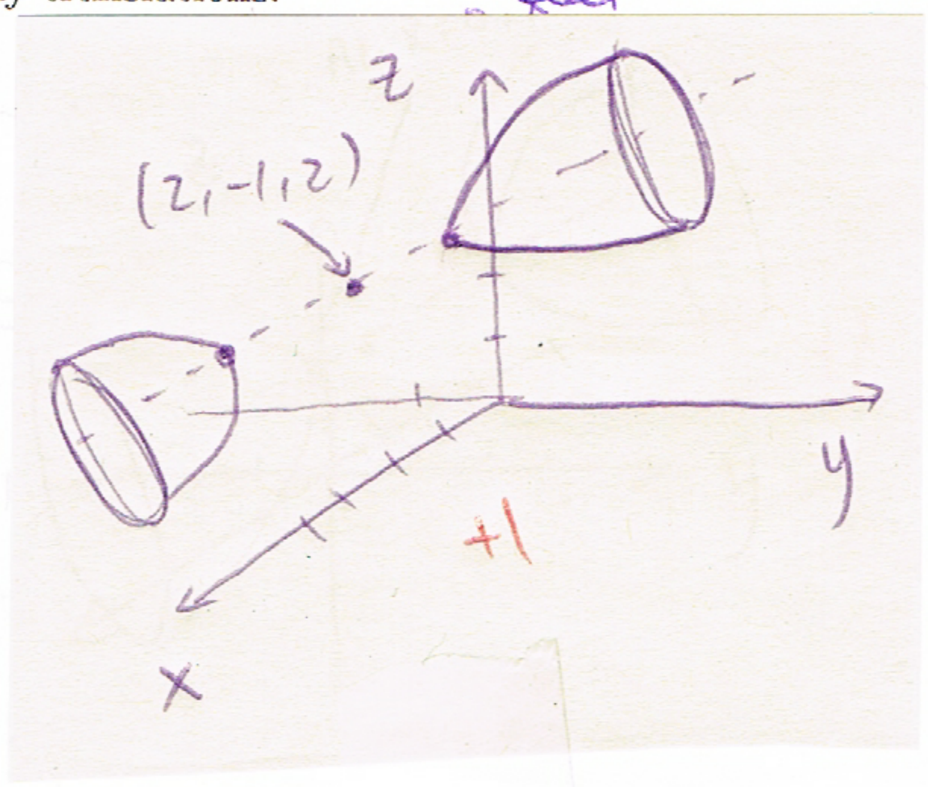
1st need to complete squares:

$(x-2)^2 - (y+1)^2 - (z-2)^2 - 5 = 4 - 1 - 4$

$= -1$ +1

We get $\boxed{(x-2)^2 \ominus (y+1)^2 \ominus (z-2)^2 = 4}$

↳ Hyperboloid of 2 sheets in x-direction



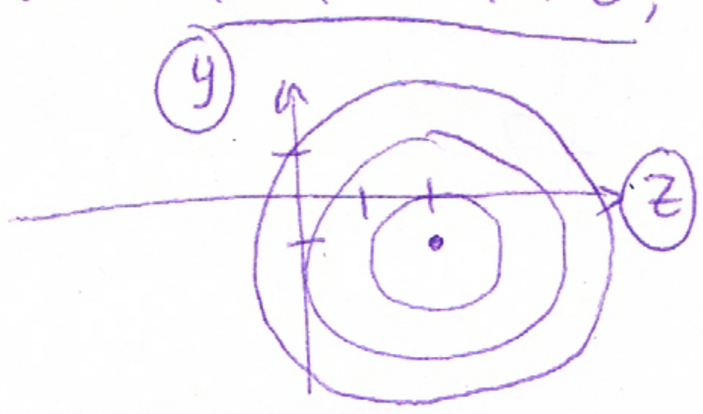
(b) Also, sketch the traces in $x = k, y = k, z = k$ respectively.
(You don't have to exactly translate your traces to your sketch; it's just for you to check part (a).)

$x \equiv k$: $(k-2)^2 - (y+1)^2 - (z-2)^2 = 4$

$\Rightarrow (y+1)^2 + (z-2)^2 = (k-2)^2 - 4$, circles

we see $k \geq 4$, $k \leq 0$ to make sense! +1

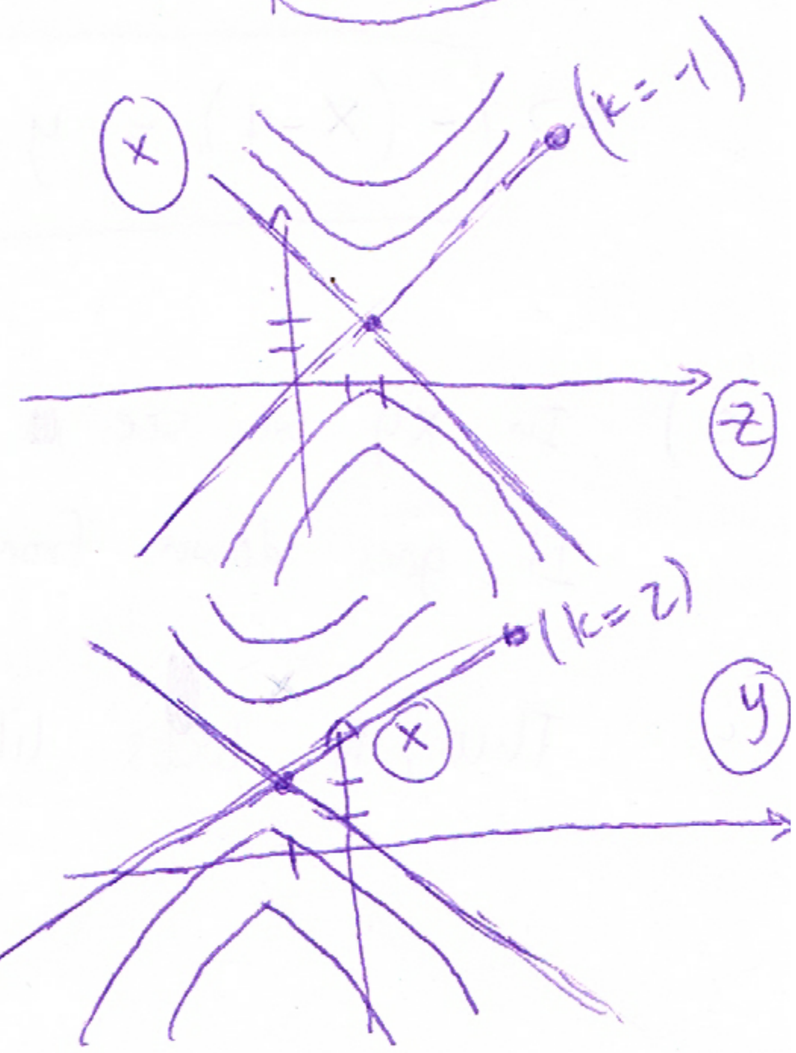
when $k \geq 4$ or $k \leq 0$,



$y \equiv k$: $(x-2)^2 - (k+1)^2 - (z-2)^2 = 4$

$\Rightarrow (x-2)^2 - (z-2)^2 = 4 + (k+1)^2$, hyperboloids

+1



$z = k$: $(x-2)^2 - (y+1)^2 - (k-2)^2 = 4$

$(x-2)^2 - (y+1)^2 = 4 + (k-2)^2$, hyperboloids

+1

2. Let a curve be parameterized by $x = e^{-t}\cos(t),\ y = e^{-t}\sin(t),\ z = e^{-t}$.
(a) Find the unit tangent vector $\mathbf{T}$ at the point (1,0,1).
(b) Find the symmetric equation of the tangent line at the point (1,0,1).
(c) **Optional** Graph the curve for $t \geq 0$. If you lose any credit from (a) and (b), part (c) can give you 1 extra credit point. Indicate the direction with arrows as t increases with arrows.

a). $\vec{r}\,'(t) = \langle -e^{-t}\cos t - e^{-t}\sin t,\ -e^{-t}\sin t + e^{-t}\cos t,\ -e^{-t} \rangle$.

At (1,0,1), we see $t = 0$. (Plug in to check) Mainly for $z = e^{-t} = 1$, $t = 0$.

So, $\vec{r}\,'(0) = \langle -1, +1, -1 \rangle \Rightarrow \boxed{\vec{T} = \dfrac{\langle -1,1,-1 \rangle}{\sqrt{1+1+1}} = \dfrac{1}{\sqrt{3}}\langle -1,1,-1 \rangle}$

+3

b). It may help to read the parametric eqns 1st,

$\Rightarrow$ For tangent line, $x = 1 - t$ (or $x = 1 - \frac{t}{\sqrt{3}}$)

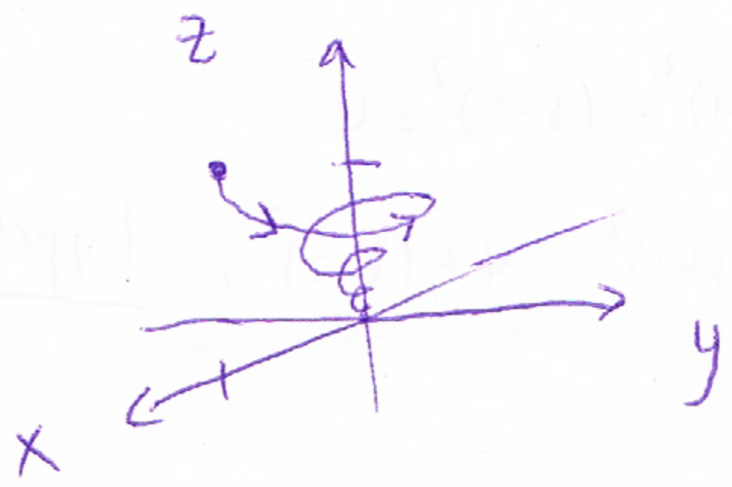
+2 $y = t$ (or $y = \frac{t}{\sqrt{3}}$)

$z = 1 - t$ (or $z = 1 - \frac{t}{\sqrt{3}}$).

$\Rightarrow \boxed{-(x-1) = y = -(z-1)}$ (or $-\sqrt{3}(x-1) = \sqrt{3}y = -\sqrt{3}(z-1)$)

c) In xy we see a spiral inwards (like circles with decaying radii)
It goes down from $z=1$ to $z=0$

Thus, it looks like



(+1 EC)